AIAA JOURNAL
Vol. 32, No. 3, March 1994

Design Optimization Methods for Rectangular Panels with Minimal Sound Radiation

John S. Lamancusa*
Pennsylvania State University, University Park, Pennsylvania 16802
and
Hans A. Eschenauer†
University of Siegen, Siegen 57068, Germany

Numerical methods are commonly used to calculate the noise radiation from vibrating structures to evaluate the acoustic performance of a particular design. Improvement of the design is then often achieved through trial and error. Although automated numerical optimization techniques are extensively utilized in some fields of structural design, their application to acoustic problems has been limited. The design of radiating structures for minimal sound radiation is a multidisciplinary problem that involves large numbers of design variables, complex objective functions, and computationally expensive computations. In this paper, methods for the design optimization of "quiet" structures using mathematical programming techniques are investigated. The objective of the study is to determine effective, general design methods for determining the optimal design of flat plates with clamped edges that minimizes the total radiated acoustic power. Variable thickness of isotropic structures and variable composite material property distributions are considered. Acoustic response is minimized either at a single frequency or over a wide frequency bandwidth. Radiated sound power is calculated using a discretized Rayleigh integral formulation, in conjunction with a finite element solver SAP4 for the solution of the structural problem. The structural analysis program and optimization procedure (SAPOP) is used.

I. Background

A. Introduction

RECENT advances in acoustic analysis, materials science, and computing technology are gradually changing the process of designing quiet structures. Where designers were once forced to rely heavily on past experience, art, and sometimes a little luck, a more organized, scientific approach is now possible. New structural and acoustic analysis techniques now make it possible to predict the acoustic performance of complicated mechanical structures before fabrication. Recent advances in computing resources and optimization procedures have made numerical optimization feasible for complex structures with many design variables. However, the design of structures for minimal sound radiation is a multidisciplinary problem that involves large numbers of design variables, complex objective functions, and computationally expensive calculations. These qualities can pose severe obstacles to the successful use of numerical optimization techniques.

The objective of this paper is to develop and to demonstrate methods for automated optimal design of acoustically radiating structures. Potential structures that can benefit from this optimal design procedure to reduce noise include lightweight noise barriers, automobile engine cover panels, and aircraft fuselage panels. With very slight modifications, the same procedures may also be used to maximize the performance of systems where acoustic radiation is desirable, such as audio loudspeakers and acoustic drivers.

B. Literature

Shape, sizing, and topology optimization techniques are becoming well-known tools, particularly for the design of static structures where the quantity of interest is minimum deflection, stress, or mass. Good examples of these methods may be found in the design of aircraft structures.[1–3] With the advent of more powerful personal computers it is now possible to apply these same successful techniques to problems that were previously neglected due to their computation intensity or multidisciplinary nature, such as the problem of structural-acoustic design.

The design of quiet plate and shell structures requires consideration of both the vibrational and acoustic responses. Several references in the literature have addressed the vibrational response of structures. These include a minimum cost design of a rectangular plate under acoustic and blast loading reported by Rao.[4] Yildiz and Stevens[5] maximized the system loss factor of a plate by optimum thickness distribution of an unconstrained visco-elastic damping laminate. References 6 and 7 have employed a technique that endeavors to remove natural frequencies from undesirable frequency bands of lightly damped structures. The material distribution of flat plates or beams for controlling the frequency of the first structural mode was addressed in Refs. 8 and 9. Placement of nodal lines as a design objective by optimal mass distribution was addressed by Ref. 10.

Shape optimization techniques have also been extended recently to coupled structural-acoustic problems. The transmission loss of a sandwich panel was optimized in Refs. 11 and 12. Several objective functions are evaluated and the authors point out that the choice of objective functions is crucial to the success of the optimization. Finite element analysis and a Rayleigh integral formulation have been used without optimization to evaluate the noise radiation from engine rocker covers.[13] Shape optimization has also been applied to the acoustic design of automobile intake and exhaust systems.[14,15] The efficiency of several structural-acoustic optimization formulations for rectangular flat panels using thickness distribution is presented in Ref. 16.

II. Method

Thin plate or shell elements are a basic component of many noise-producing structures and are therefore an important element in any strategy for the design of quiet structures. In this study, the optimal thickness or mass distribution of flat plates is determined, which minimizes the total radiated sound power over a wide fre-

Presented as Paper 92-4768 at the AIAA/USAF/NASA/OAI 4th Symposium on Multidisciplinary Analysis and Optimization, Cleveland, OH, Sept. 21–23, 1992; received Jan. 15, 1993; revision received Aug. 27, 1993; accepted for publication Aug. 27, 1993.

*Associate Professor, Department of Mechanical Engineering, Center for Acoustics and Vibrations, 157 Hammond Building.

†Director, Research Laboratory for Multidisciplinary Analysis and Applied Structural Optimization.

quency bandwidth. The case considered is a flat, rectangular plate that has built-in edges and is excited by a uniform force. The formulation presented here is general in nature and is applicable to the optimal design of any structure whose dynamic response can be accurately analyzed by finite element analysis, and whose acoustic radiation can be predicted using the Rayleigh integral.

The required equations for calculation of the structural and acoustic responses, as well as a brief description of acoustic radiation phenomena, are presented in the following sections.

A. Structural Analysis

Finite element analysis is used to solve the structural vibration problem. The eigenvalue analysis of the plate is performed using the SAP4 program.[17] This program was chosen over some newer commercial alternatives because of the availability of its source code and thereby the ability to merge it easily with optimization algorithms within a larger program structure.

Standard linear modal superposition techniques are used to calculate the surface velocity at the nodal points of the structure from the eigenvectors, assuming proportional damping (a modal damping ratio of ξ). The mean square normal velocity of the plate $\langle \tilde{v}_n^2 \rangle$ is obtained by averaging the surface normal velocity $\tilde{v}_i$ over all n node points of the structure:

$$\langle \tilde{v}_n^2 \rangle = \frac{1}{n} \sum_{i=1}^{n} |\tilde{v}_i|^2 \tag{1}$$

B. Acoustic Analysis

The acoustic radiation from a planar surface in an infinite baffle may be calculated once the surface normal velocities are known, by use of the Rayleigh integral formulation[18]:

$$p(\boldsymbol{r}) = \frac{j\omega\rho}{2\pi} \int_s \frac{\tilde{v}(\boldsymbol{r}_s)\, e^{-jkR}}{R}\, \mathrm{d}S \tag{2}$$

where

$p(\boldsymbol{r})$ = complex sound pressure at $\boldsymbol{r}$
$\boldsymbol{r}$ = position of observation point
$\boldsymbol{r}_s$ = position of elemental surface δS
$R = |\boldsymbol{r} - \boldsymbol{r}_s|$
$\tilde{v}(\boldsymbol{r}_s)$ = normal velocity of δS
k = acoustic wave number (ω/c)
ω = frequency, rad/s
ρ = mean density of fluid
c = sound velocity of fluid
$j = \sqrt{-1}$

This expression results in a mathematical singularity when evaluating the pressure on an element due to its own velocity since $R = 0$. The "self-contribution" of a rectangular element having uniform velocity over its surface may be determined by evaluating the singular integral over the element area:

$$\int \frac{1}{R}\, \mathrm{d}S = 2a \ln \left[\frac{\sqrt{a^2+b^2}+b}{\sqrt{a^2+b^2}-b} \right] + 2b \ln \left[\frac{\sqrt{a^2+b^2}+a}{\sqrt{a^2+b^2}-a} \right] \tag{3}$$

where $2a$ and $2b$ are the length and width of the surface element, respectively. For the entire surface, the pressure is evaluated at the centroid of the mth element having area A, by summing the contributions due to the average normal velocity $\bar{v}_l$ (the average of the four nodal point velocities for the lth element) of all n elements using the discretized form of Eq. (2):

$$p_m = \frac{j\omega\rho}{2\pi} \left[\sum_{l=1}^{n} \frac{\bar{v}_m e^{-jkR_{l,m}} A_l}{R_{l,m}} + \bar{v}_m \int \frac{1}{R_m}\, \mathrm{d}S \right] \quad \text{for } l \neq m \tag{4}$$

where the second term in the equation represents the pressure at the center of the element due to its own velocity, calculated using Eq. (3).

The acoustic intensity I is evaluated on the surface of the plate by:

$$I(\boldsymbol{r}_s) = \tfrac{1}{2} \mathrm{Re}\,[p(\boldsymbol{r}_s)\, \tilde{v}^*(\boldsymbol{r}_s)] \tag{5}$$

where Re denotes the real part of a complex quantity and * the complex conjugate.

The total radiated sound power W is obtained by integrating the intensity over the surface S:

$$W = \int_s I(\boldsymbol{r}_s)\, \mathrm{d}S \tag{6}$$

An alternative method for obtaining sound power is to integrate the sound intensity over a hemisphere in the far field. However, integration over the surface is more computationally efficient than integration over a hemisphere, except for low values of ka where directionality is negligible. Surface integration requires sound pressure calculations at far fewer points than integration over a hemisphere in the farfield.

In discretized form for numerical computation, the power is summed over each of the n total surface elements:

$$W = \tfrac{1}{2} \mathrm{Re} \left[\sum_{m=1}^{n} A_m \bar{p}_m \bar{v}_m^* \right] \tag{7}$$

where $\bar{v}_m$ is the average velocity over the mth element, $\bar{p}_m$ is the average sound pressure amplitude, and A_m is the element area. This discretized approach provides good results provided that the size of a surface element does not exceed approximately one-quarter of an acoustic wavelength.

A commonly used measure of the sound radiation characteristics is the radiation efficiency σ. This is defined as the ratio of power radiated by the structure in question to that of a uniformly vibrating baffled circular piston of the same area S and with velocity equal to the average mean square velocity of the structure in question ($ka \gg 1$):

$$\sigma = \frac{W}{\tfrac{1}{2}\rho c S \langle \tilde{v}_n^2 \rangle} \tag{8}$$

The structural and acoustic equations were implemented in a Fortran program using the Lahey F77L-EM/32 compiler for execution on a 33-MHz 486 personal computer. The validity of the coupled vibration-acoustic calculation was verified by comparison to the closed-form solution for a simply supported square plate by Wallace.[19]

A qualitative understanding of the physics of any problem is an essential prerequisite to successful numerical optimization. The computed radiation efficiency for the first six modes of a rectangular plate with clamped edges is shown in Fig. 1. The coincidence, or critical frequency ω_c, is the frequency at which each individual mode begins to radiate efficiently ($\sigma \approx 1$) and is defined as the frequency at which the structural wavelength equals the acoustic wavelength. Below the coincidence frequency, the radiation of acoustic energy is very inefficient:

$$\omega_c = c \sqrt{\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2} \tag{9}$$

where c is the sound velocity in air, p and q are the mode indices (essentially the number of half sine waves in each direction on the plate), a is the plate length in the same direction as p, and b is the plate length in the direction of q.

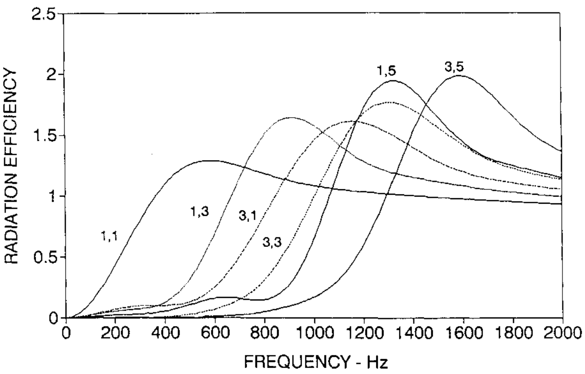

Fig. 1 Calculated radiation efficiency of first six symmetric modes with mode indices (p, q) for a 600 × 800-mm clamped-edge plate.

The critical frequencies for each individual mode are independent of plate thickness and are listed in Table 1. Also listed are the natural frequencies and radiation efficiencies of the first eight modes of a thin (4.92 mm) and a thick (12.3 mm) uniform thickness plate with clamped edges. Since modes with natural frequencies below the critical value have radiation efficiencies less than one, the thin plate exhibits low radiation efficiency in each mode since each mode is below its critical frequency. All of the modal frequencies of the thicker plate except the fundamental are higher than the critical frequency, resulting in values of radiation efficiency approaching unity.

The frequency response of a uniform thickness (4.92 mm), clamped-edge flat steel plate with dimensions (600 × 800 mm) in response to a uniformly distributed force, is shown in Fig. 2. This figure illustrates the fundamental acoustic behavior of a vibrating plate and the acoustic superposition effects that occur when more than one mode is excited. The natural frequencies of the plate can plainly be seen as peaks in both the power and the mean square velocity curves. The "waves" in the radiation efficiency curve are due to the superposition of the mode shapes and efficiencies of the individual modes. At low frequencies, the radiation efficiency is small and the plate's vibrational energy is poorly coupled into acoustic radiation. However, at high frequencies the efficiency tends toward unity, and the sound power is proportional to the mean square velocity.

From the preceding equations, a number of qualitative observations can be made that are useful to keep in mind when formulating the optimization problem and when reviewing the optimization results. These include the following:

1) Minimizing the mean square velocity of the plate will decrease the radiated power of a structure.

2) To decrease the radiation efficiency of a given mode shape, its natural frequency should be as far below its coincidence frequency as possible.

3) Above critical frequency, radiated power is proportional to mean square velocity.

4) At any frequency below coincidence, a modal pattern with as many zero crossings (or nodal lines) as possible is most desirable for minimum radiation efficiency.

III. Mathematical Programming Problem Formulation

A. Problem Definition

The minimization of radiated sound power from a vibrating structure is a multidisciplinary, computationally intensive problem having the potential for large numbers of design variables and constraints. Mathematical programming techniques were chosen for this study due to their flexibility in formulation of objective functions and constraints and the existence of several proven optimization algorithms.

Following standard notation, the general form of the nonlinear programming problem may be stated as follows[20]:

Find a vector of design variables

$$\boldsymbol{b} = (b_1, b_2, \ldots, x_{ndv})$$

which minimizes the cost or objective function

$$f(\boldsymbol{b})$$

subject to equality constraints

$$h_i(\boldsymbol{b}) = 0 \qquad i = 1, \ldots, p$$

and inequality constraints

$$g_j(\boldsymbol{b}) \leq 0 j = 1, \ldots, m$$

Upper and lower bounds on the design variables also may be imposed such that $b_i^{\text{lower}} \leq b_i \leq b_i^{\text{upper}}$. Mathematically and numerically, these are usually treated like inequality constraints. The structural analysis program and optimization procedure (SAPOP) program[21] was chosen for this problem due to its proven efficiency and utility. This package features a number of optimizer algorithms that can be applied interchangeably to suit the particular demands of a given problem. The particular optimizer used in this study used the hybrid SQP-GRG algorithm,[22] which employs the sequential quadratic programming (SQP) search direction and the generalized reduced gradient (GRG) line search. This algorithm features good convergence and is applicable to nonlinear problems with large numbers of variables and/or constraints. Other optimizers in the SAPOP package include a sequential linear programming (SLP), SQP, and GRG. For performance comparison, the

Table 1 Cut-on frequencies for plate modes and natural frequencies f_n and radiation efficiency values σ for the first eight modes of 600 × 800-mm constant thickness (4.92 and 12.3 mm) plates with clamped edges

		h = 4.92 mm		h = 12.3 mm	
Mode (p, q)	Critical f_c, Hz	f_n, Hz	σ	f_n, Hz	σ
1, 1	355	110	0.14	270	0.68
1, 3	705	305	0.07	750	1.34
3, 1	885	480	0.11	1185	1.61
3, 3	1070	665	0.08	1625	1.39
1, 5	1110	685	0.14	1680	1.34
3, 5	1375	1030	0.13	2520	1.08
5, 1	1445	1170	0.17	2860	1.00
5, 3	1565	1245	0.03	3035	.98

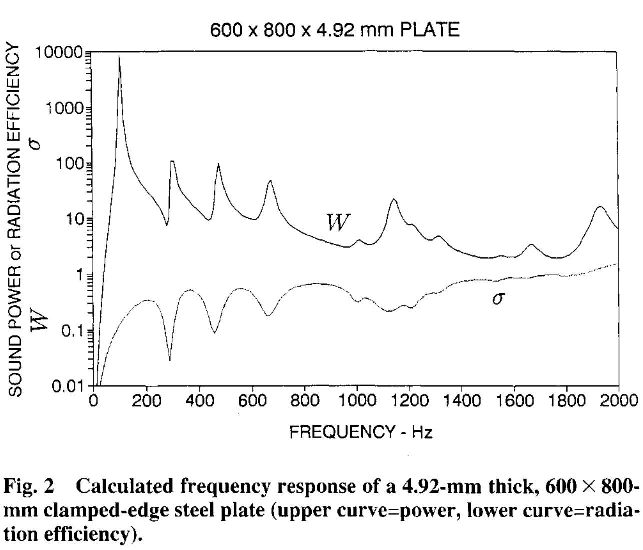

Fig. 2 Calculated frequency response of a 4.92-mm thick, 600 × 800-mm clamped-edge steel plate (upper curve=power, lower curve=radiation efficiency).

CONMIN program,[23] which uses the method of feasible directions, was also used for a few cases.

The choice of design variables, constraints, and objective function specific to the acoustic problem are described next.

B. Design Variables

The dynamic performance of an isotropic structure depends on the distribution of mass, stiffness, and damping. These properties can be altered by addition of lumped masses, layered damping materials, and changes in geometry such as thickness. The actual choice of design variables is dictated by the specific performance and manufacturing realities of the problem to be solved. Panels with continuous or stepped thickness variation may be appropriate and feasible in some cases, such as in molded plastic or machined metal parts. With stamped sheet metal parts, the placement and size of formed bends or stamped-in stiffening ridges would be a natural choice for design variables. Weldments offer the ability to add stiffening ribs or lumped masses. The location and size of surface-mounted constrained-layer damping treatments are also viable choices in some instances. The use of fiber-reinforced composite materials introduces additional parameters that may be used as design variables such as fiber ply angles, ply thicknesses, and fiber volume fraction. In this study, thickness and mass distribution are used as design variables for isotropic plates, and volume fraction distribution is used for a fiber-reinforced composite sandwich plate.

1. Thickness Distribution

The isotropic plates in this study are modeled using the SAP4 thin shell element. This element is a constant thickness, plane stress element with four nodes and six degrees of freedom per node, which is capable of orthotropic behavior.[17,24]

The thickness of each shell element can be used as a design variable as in Ref. 16. However, for models with a large number of elements, this can easily result in far too many variables for efficient numerical optimization. This can also result in highly irregular structures, with large thickness variations from element to element. Higher order finite elements, which allow thickness change across an element, may also be used at the expense of additional degrees of freedom per element.

A parametric description of thickness, such as a bicubic polynomial, or a bicubic spline may be used to reduce the total number of thickness variables and to smooth the resulting optimized surface. In the polynomial form, the thickness distribution as a function of x, y position on the plate, $h(x, y)$, may be described by a polynomial of order N:

$$h(x, y) = \sum_{i=0}^{N}\sum_{j=0}^{N} B_{ij} x^i y^i \tag{10}$$

where B_{ij} are constant coefficients that become the design variables for the optimization. The thicknesses of each finite element

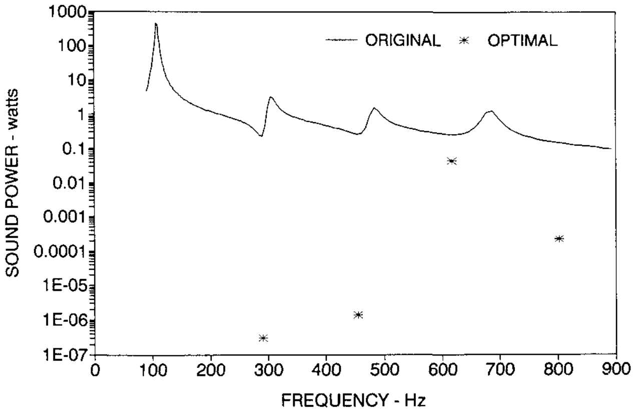

Fig. 3 Single frequency optimization results for rectangular plate (initial thickness 4.92 mm) thickness distribution.

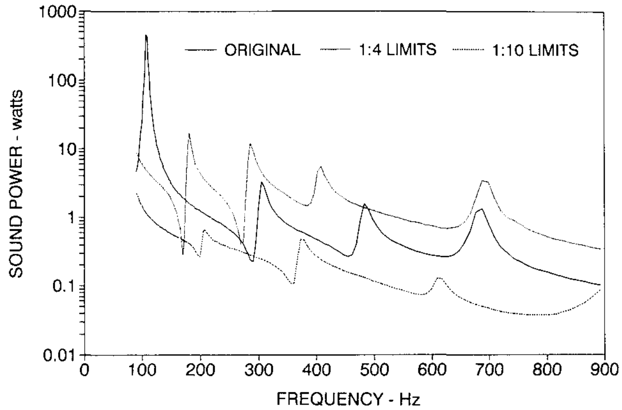

Fig. 4 Broadband frequency optimization results for rectangular plate (initial thickness 4.92 mm) thickness distribution for a) 1:4 bounds on thickness variables and b) 1:10 bounds on thickness variables.

are calculated at their centroid using Eq. (10). Upper and lower bounds on thickness are imposed using inequality constraints of the form

$$g_i = \frac{z_i}{z_{\max}} - 1 \quad \text{and} \quad g_i = \frac{z_{\min}}{z_i} \tag{11}$$

A bicubic spline is used in this study to interpolate thickness values from a set of specified spline points.[25] The thickness at each spline point becomes the design vector $\boldsymbol{b}$. This technique has the advantage over the polynomial form in that simple upper and lower bound constraints can be placed directly on the spline point thicknesses, since the fitted surface must pass through those points. An additional advantage of using the spline over the polynomial form is that each design variable is of the same order of magnitude, and no rescaling is necessary.

2. Composite Material Property Variation

A fiber-reinforced composite sandwich construction is modeled with SAP4. This construction consists of a low modulus, lightweight core with a low shear modulus (such as an aluminum honeycomb), and balanced face plies of graphite/epoxy laminates. A three-dimensional solid element with eight nodes per element is used to model the core material. Thin shell elements are used to model the surface plies. These face plies are constructed in two layers, one layer with fibers running in the x direction and the second with fibers running in the y direction. Each layer is composed of parallel strips of equal width. Within each strip the volume fraction is constant, but each strip can have a different volume fraction. The volume fractions of the strips are the design variables for optimization. The width of each strip is the same as the width of the finite element. For an $n \times m$ finite element mesh, this results in $n + m$ design variables.

The stress-strain relations for an orthotropic material with fibers running in the 11 directions are[26,27]

$$\{\sigma\} = [C]\{\epsilon\}$$

$$\{\sigma\} = \{\sigma_1 \ \sigma_2 \ \tau_{12}\} \tag{12}$$

$$\{\epsilon\} = \{\epsilon_1 \ \epsilon_1 \ \gamma_{12}\}$$

$$[C] = \frac{1}{(1-\nu_{12}\nu_{21})}\begin{bmatrix} E_{11} & \nu_{21}E_{11} & 0 \\ \nu_{21}E_{11} & E_{22} & 0 \\ 0 & 0 & (1-\nu_{12}\nu_{21})G_{12} \end{bmatrix}$$

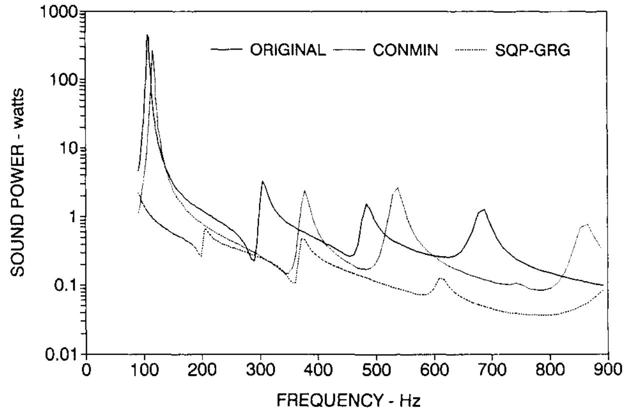

Fig. 5 Comparison of broadband optimal designs using CONMIN and SQP-GRG algorithms.

The individual terms of the $[C]$ matrix:

$$E_{11} = E_f V_f + (1 - V_f)E_m \tag{13}$$

$$E_{22} = \frac{E_f E_m}{(1 - V_f) E_f + V_f E_m} \tag{14}$$

$$\nu_{12} = (1 - V_f)\nu_m + V_f \nu_f \tag{15}$$

$$G_{12} = \frac{G_m G_f}{(1 - V_f) G_f + V_f G_m} \tag{16}$$

$$\frac{\nu_{21}}{E_{22}} = \frac{\nu_{12}}{E_{11}} \tag{17}$$

where f and m denote properties of the fiber and matrix, respectively; E is the elastic modulus; G is the modulus of rigidity; and V_f is the fiber volume fraction. Similar relations can be written for fibers running in the 22 direction.

The combined stress-strain relation matrices for the two equal thickness layers, one with fibers running in the x direction, and the second with fibers in the y direction, are found by averaging the contribution of each:

$$[C] = \tfrac{1}{2}\{[C]_x + [C]_y\} \tag{18}$$

where $[C]_x$ and $[C]_y$ refer to the individual matrices for the layers with fibers running in the x (11) and y (22) directions, respectively.

C. Objective Functions

Numerous possibilities exist for formulating a performance measure suitable for use as an objective function, $f(\boldsymbol{b})$. A previous paper examined the acoustic optimization problem formulation and identified effective objective functions for minimization.[16] The choice of an appropriate objective function is itself a compromise between speed and accuracy. Three objective function formulations are examined in this study and are described next.

1. Radiated Sound Power

The average sound power over a frequency bandwidth is calculated by integrating the acoustic power [(Eq. 7)] over the desired frequency interval $f_{\text{span}} = f_{\max} - f_{\min}$:

$$f_1 = \hat{W} = \frac{1}{f_{\text{span}}} \int_{f_{\min}}^{f_{\max}} W \, \mathrm{d}f \tag{19}$$

The integral is evaluated by evaluating power at n_{freq} discrete frequencies and using a trapezoidal rule approximation.

In this study, either single frequency excitation (and single frequency response) or multiple frequency excitation (and banded frequency response) were considered. Several frequencies were examined for the single frequency case. For the banded frequency case, 150 frequency increments were used over a frequency range of one decade. Lower and upper bound constraints were placed on the design variables.

In many instances, it may be desirable to specify, in addition to upper and lower bounds on the thickness variables, a maximum plate mass. This is accomplished using an inequality constraint of the form

$$g_1(\boldsymbol{x}) = -1 + \frac{m_{\text{plate}}(\boldsymbol{x})}{m_{\max}} \tag{20}$$

where $m_{\text{plate}}(\boldsymbol{x})$ is the calculated mass for the present design, and $m_{\max}$ is the specified mass limit. This constraint is linear with respect to the design variables and is considered to be violated when its value becomes positive.

2. Radiated Sound Power with Constant Mass

One physically intuitive approach to varying the plate geometry, while maintaining a constant mass, is that any material that is added to one plate element must be taken from another element on the plate. The receiving element must have a positive sensitivity (power decreases with increasing thickness), whereas the "donor" element must have a less positive (or negative) sensitivity for a net power reduction to occur. A new design variable a_i is used that describes the material sharing between a pair of elements:

$$x_i^h = a_i x_0 \quad \text{and} \quad x_i^l = (2 - a_i)x_0 \quad i = 1, \ldots, n_{dv}/2 \tag{21}$$

where x^h is the thickness of the more sensitive element, x^l is the thickness of the less sensitive element, n_{dv} is the original number of design variables (the number of finite elements), and x_0 is the initial thickness of the elements (and also the average thickness).

The two elements in each pair are chosen after an initial sensitivity analysis on all of the thickness variables. The variables are then rank ordered (rank = r) and paired off, x^r to $x^{r+n_{dv}/2}$ for r =

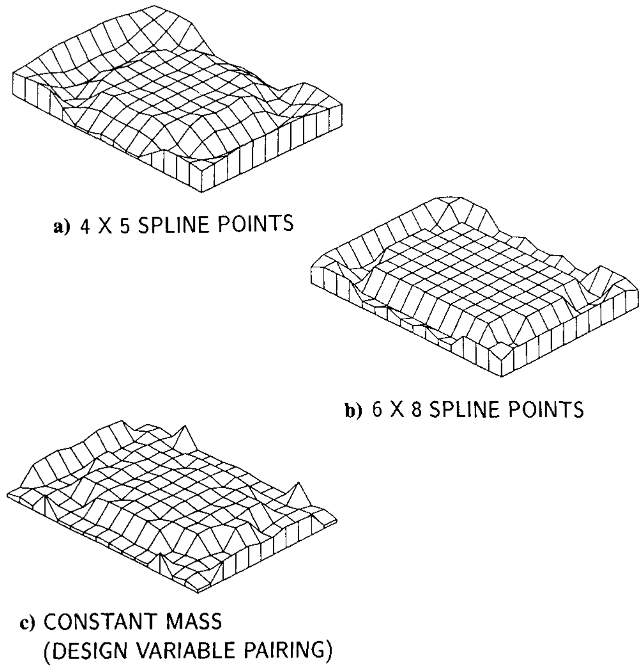

Fig. 6 Optimum thickness distribution for 600 × 800-mm plate to minimize power from 90–900 Hz: a) 4 × 5 spline points, b) all variables (48), and c) constant mass objective.

$1, \ldots, n_{dv}/2$. A new sensitivity analysis is then performed on the reduced design variables a_i, and the optimization then commences. Besides guaranteeing constant mass, this formulation also has the advantage of halving the number of design variables. This simplifies the optimization process and halves the time spent performing sensitivity analyses. Upper and lower bounds are placed on the sharing coefficients a_i to maintain the actual thicknesses within specified limits. This formulation implicitly results in thickness limits that are symmetric about the initial thickness value. The objective function in this case is also the radiated acoustic power, calculated as in Eq. (19).

3. Minimum Mass with Upper Bound Power Constraint

All of the objective functions mentioned up to this point are highly nonlinear with respect to the design variables; however, the mass constraint is a linear function. The use of mass as an objective function, with power included as a constraint, may potentially result in a more well-formed problem for some optimization algorithms. The objective function in this instance is the total mass m of the structure:

$$f_3 = m = \sum_{i=1}^{n_{el}} x_i A_i \rho \tag{22}$$

and the inequality constraint that limits the maximum power is

$$g_3(x) = -1 + \frac{\hat{W}}{\hat{W}_{max}} \tag{23}$$

where ρ is the mass density, A is the element area, n_{el} is the total number of elements, and $\hat{W}_{max}$ is the maximum allowable sound power.

Although the objective function is now linear with respect to the design variables, the inequality constraint is highly nonlinear. This may adversely affect the efficiency of some optimization algorithms. The CONMIN algorithm, for instance, could not find a good optimal solution in this case (it could not find a solution that was significantly better than the initial constant thickness design), whereas the SQP-GRG algorithm had no problem in finding an optimal solution.

4. Other Objective Functions

Historically, some of the first efforts to optimize dynamic performance did so by maximizing the fundamental frequency of the structure, to force the lowest resonance to be above an objectionable frequency range. This can be formulated as a conventional minimization problem by using the inverse of the fundamental frequency as the objective function. Other choices for objective functions include minimizing the radiated power at the resonant frequencies, minimizing the mean square velocity, minimizing the radiation efficiency, or forcing the structure to vibrate in a predetermined modal pattern.

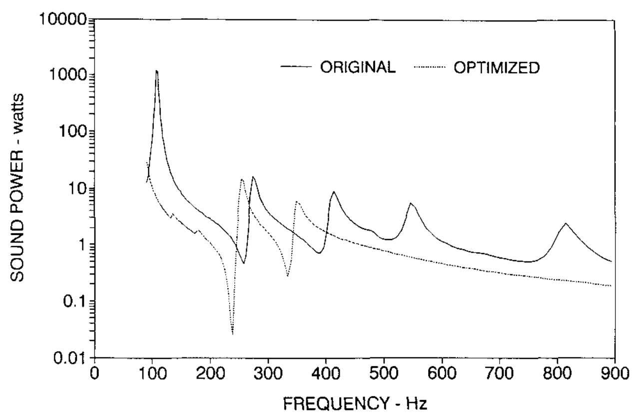

Fig. 7 Frequency response of original design of sandwich composite plate (constant V_f = 0.40) and optimized design.

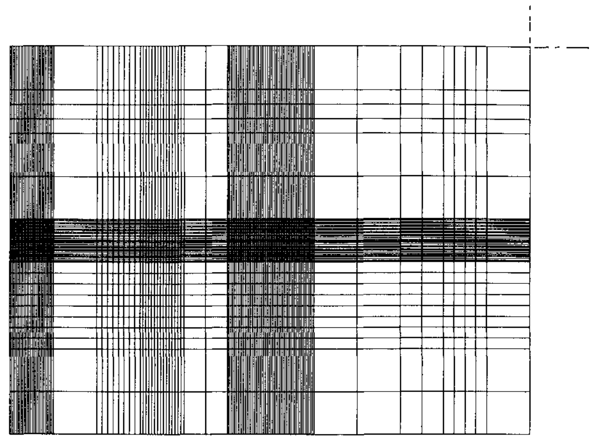

Fig. 8 Optimum volume fraction distribution for one-quarter of a 600 × 800-mm plate that minimizes the radiated power from 90–900 Hz (center of plate is upper right corner).

D. Choice of Constraints

The numerical values of the lower and upper bounds on design variables depend on the actual problem to be solved. In practice, the physical bounds on design variables are a function of the material and the manufacturing process to be used. In addition, other factors, such as maximum allowable stress or maximum deflection, may limit the design space. Limits on element aspect ratio or thickness variation from element to element may also be necessary to insure modeling accuracy, depending on the boundary conditions and the type of finite element used in the analysis. In general, any optimization procedure must have sufficient design freedom to find an acceptable solution. Typical optimizers work more efficiently with wide bounds on design variables and as few inequality constraints as possible.

IV. Applications

A. Problem Description

A flat rectangular plate with a 3:4 aspect ratio and dimensions of 600 × 800 mm was studied. Two optimization approaches are employed: 1) determine the optimal thickness distribution of an isotropic (steel) plate, and 2) determine the optimal volume fraction distribution of a sandwich composite plate.

Each case was designed to be dynamically similar, with the initial designs all having the same natural frequency of 108 Hz. For the broadband cases, a 90–900 Hz frequency band was used, which encompasses the first five symmetric modes of each plate.

Symmetry was employed in the finite element analysis to reduce the required number of elements. One-quarter of the plate was modeled by 48 elements (6 × 8 mesh) for the isotropic material cases and 84 elements (9 × 12 mesh) for the composite material plate. Each element was described by a constant thickness. A zero slope boundary condition was enforced along the two edges of the model that correspond to the centerline of the plate. As a result of the assumed symmetry, only the modes that are symmetric about the centerlines of the plate are calculated. In acoustic terminology, these are called the nonsymmetric or odd modes because they have an odd number of half sine waves in their mode shapes. These modes are the most significant contributors to acoustic radiation because they have the least amount of inherent self-cancellation at frequencies well below coincidence.[18] The mesh refinements were chosen as compromises between the mutually exclusive goals of modeling accuracy (both structural and acoustic) and minimizing the number of elements. A modal damping ratio of $\xi_i = 0.02$ was found from experiments to be representative of a flat plate with clamped edges. The material properties for each case are listed in Table 2.

Table 2 Material properties used in analyses

Material property	Isotropic plate	Composite	
		Fiber	Matrix
E modulus, GPa	207	230	3.5
Density, kg/m^3	7.85E3	1.20E3	1.74E3
Poisson ratio, ν	0.30	0.23	0.30
G modulus, GPa	79	50	1.35

All of the optimizers used in this study require the user to provide sensitivity information at each design step. This was determined using divided difference (a 0.05-mm perturbation of each design variable was used).

B. Thickness Distribution Results

The acoustic performance as a function of frequency of a steel plate of constant thickness 4.92 mm is shown in Fig. 3. Single frequency optimizations using the SQP-GRG algorithm were performed at frequencies corresponding to the minimums of the frequency response curve (290, 450, 620, and 800 Hz). Lower and upper thickness bounds of 0.89 and 8.94 mm (1:10 ratio) and an initial thickness of 4.92 mm were specified. After optimization, the sound power was reduced by 59, 53, 18, and 28 dB, respectively, for the four frequencies as shown in Fig. 3.

The broadband power from 90–900 Hz was then minimized using two different ranges of lower and upper bounds, 1:10 (0.894–8.94 mm) and 1:4 (1.97–7.87 mm). The results are displayed in Fig. 4 and demonstrate that wider bounds typically result in lower optimums (13.2 vs 3.4 dB improvement). These cases resulted in increases in the plate weight of 30 and 27%.

A comparison between the optimums found by the CONMIN algorithm (2.1-dB improvement) and the SQP-GRG algorithm (13.2-dB improvement) is shown in Fig. 5. The use of fewer design variables (4×5 spline points instead of all 6×8 points) resulted in an improvement of 3.7 dB. The thickness distributions determined by the SQP-GRG algorithm for the cases with 4×5 spline points and all 6×8 spline points are shown in Fig. 6.

The constant mass objective function formulation (Sec. III.C.2) resulted in a power reduction of 8.4 dB and the thickness distribution shown in Fig. 6c. The objective function of minimizing the plate mass (Sec. III.C.3), while maintaining the same radiated power, was also effective and resulted in a mass reduction of 69%.

C. Composite Material Distribution Results

A sandwich composite plate with a 10-mm-thick core layer and 2-mm-thick graphite/epoxy surface plies was optimized for minimum power at single frequencies and over a broad frequency bandwidth. The starting point of the optimization was an initial design with all volume fractions equal to 0.40. Lower and upper bounds of 0.0 and 0.90 were imposed. In each quarter of the plate, the face plies were divided into nine strips running in the y direction (parallel to the long side of the plate) and 12 strips running in the x direction.

A broadband optimization from 90–900 Hz resulted in a reduction of 8.8 dB. The frequency responses for the original design and the broadband optimal design are shown in Fig. 7. The volume fraction distribution of the optimal design is shown in Fig. 8, where the solid lines represent the direction of the reinforcing fibers and the density of the lines denotes the volume fraction.

V. Conclusions

It has been demonstrated that optimization of sizing and material property variables can be effectively used to minimize the radiated sound power from rectangular plates at a single frequency or over a wide frequency bandwidth. Reductions are greatest for single frequency inputs.

Several important lessons were learned, demonstrating that optimization cannot be used simply as a "black box" to solve all problems, and a fast computer does not eliminate the need to understand the underlying physics. The choice of an appropriate objective function is critical to the success of numerical optimization. Highly nonconvex objective functions may result in excessive computations and trapping in local minima. Acoustic power is the most direct measure of acoustic performance and, when used as the objective function without weight constraint, produces the most consistently improved designs. The disadvantages of using radiated power are that it is costly to compute for many frequencies and results in substantial weight changes (usually increases). The constant mass formulation is effective, cuts design variables in half, negates the need for inequality constraints, and is an effective way to insure no mass change during optimization. Mass minimization, with constraint on maximum power, is effective provided the optimizer used can handle the nonlinear constraint. For nonisotropic plates, optimal distribution of the composite material volume fraction results in substantial reductions in broadband sound radiation.

In closing, it should also be noted that the optimal solution is highly dependent on the location and distribution of excitation force, the structural finite element discretization, and the latitude given for design variable variation. The sensitivity of the final answer to the optimizer algorithm is an acknowledged limitation and unavoidable frustration of numerical optimization techniques. The specifics of each problem (degree of nonlinearity of objective function and constraint formulations) dictate how well a particular optimizer will perform. In practice, several different algorithms are typically applied to the same problem, in effect, "optimizing the optimizer."

Acknowledgment

This work was supported by the Alexander von Humboldt Foundation while the first author was on sabbatical leave at the University of Siegen, Siegen, Germany

References

[1]Eschenauer, H. A., Schuhmacher, G., and Hartzheim, W., "Optimization of Fiber Composite Aircraft Structures by Means of the Procedure Lagrange," *Computers and Structures*, Vol. 44, No. 4, 1992, pp. 877–894.

[2]Grandi, R., Bharatram, G., and Venkayya, V., "Efficient Multi-Objective Optimization Scheme for Large Scale Structures," *Proceedings of the 4th AIAA/USAF/NASA/OAI Symposium on Multidisciplinary Analysis & Optimization* (Cleveland, OH), AIAA, Washington, DC, 1992, pp. 679–685 (AIAA Paper 92-4772).

[3]Sobieszczanski-Sobieski, J., "Multidisciplinary Optimization for Engineering Systems: Achievements and Potential," *Optimization: Methods and Applications, Possibilities and Limitations*, Springer-Verlag, Berlin, 1989, pp. 42–62.

[4]Rao, S. S., "Structural Optimization Under Combined Blast and Acoustic Loading," *AIAA Journal*, Vol. 14, No. 2, 1976, pp. 276–278.

[5]Yildiz, A., and Stevens, K., " Optimum Thickness Distribution of Unconstrained Visco-Elastic Damping Layer Treatments for Plates," *Journal of Sound and Vibration*, Vol. 103, No. 3, 1985, pp. 183–199.

[6]Starkey, J. M., and Bernhard, J. E., "A Constraint Function Technique for Improved Structural Dynamics," *ASME Journal of Vibrations, Acoustics and Reliability in Design*, Vol. 108, No. 1, 1986, pp. 101–106.

[7]Watt, D., and Starkey, J., "Design Optimization of Response Amplitudes in Viscously Damped Structures," *ASME Journal of Vibrations and Acoustics*, Vol. 112, No. 3, 1990, pp. 275–280.

[8]Olhoff, N., "Optimal Design of Vibrating Rectangular Plates," *International Journal of Solid Structures*, Vol. 10, No. 1, 1974, pp. 93–109.

[9]Thambiratnam, D., and Thevendran, V., "Optimum Vibrating Shapes of Beams and Circular Plates," *Journal of Sound and Vibration*, Vol. 121, No. 1, 1988, pp. 13–23.

[10]Pritchard, J. I., Adelman, H. M., and Haftka, R. T., "Sensitivity Analysis and Optimization of Nodal Point Placement for Vibration Reduction," *Journal of Sound and Vibration*, Vol. 119, No. 2, 1987, pp. 277–289.

[11]Lang, M. A., and Dym, C. L., "Optimal Acoustic Design of Sandwich Panels," *Journal of the Acoustic Society of America*, Vol. 57, No. 6-II, 1974, pp. 1481–1487.

[12]Makris, S., Dym, C. L., and Smith, J., "Transmission Loss Optimization in Acoustic Sandwich Panels," *Journal of the Acoustic Society of America*, Vol. 79, No. 6, 1986, pp. 1833–1843.

[13]Sivakumar, J., Shung, S. H., and Nefske, D. J., "Noise Reduction of Engine Component Covers Using the Finite Element Method," *ASME Structural Vibration and Acoustics*, DE-Vol. 34, Book G0644A, Sept. 1991, pp. 133–137.

[14]Bernhard, R. J., "A Finite Element Method for Synthesis of Acoustical Shapes," *Journal of Sound and Vibration*, Vol. 98, No. 1, 1985, pp. 55–65.

[15]Lamancusa, J. S., "Geometric Optimization of Internal Combustion Engine Induction Systems for Minimum Noise Transmission," *Journal of Sound and Vibration*, Vol. 127, No. 2, 1988, pp. 303–318.

[16]Lamancusa, J. S., "Numerical Optimization Techniques for Structural-Acoustic Design of Rectangular Panels," *Computers and Structures*, Vol. 48, No. 4, 1993, pp. 661–675.

[17]Bathe, K., Wilson, E., and Peterson, F., "SAPIV, A Structural Analysis Program for Static and Dynamic Response of Linear Systems," College of Engineering, Univ. of California at Berkeley, Rept. EERC 73-11, Berkeley, CA, revised April 1974.

[18]Fahy, F., *Sound and Structural Vibration: Radiation, Transmission and Response*, Academic Press, London, 1985.

[19]Wallace, C. E., "Radiation Resistance of a Rectangular Panel," *Journal of the Acoustic Society of America*, Vol. 51, No. 2, 1972, pp. 946–952.

[20]Arora, J. S., *Introduction to Optimum Design*, McGraw-Hill, New York, 1989.

[21]Eschenauer, H. A., Geilen, J., and Wahl, H. J., "SAPOP—An Optimization Procedure for Multicriteria Structural Design," *Numerical Methods in FE-Based Structural Optimization Systems*, edited by K. Schittkowski and H. Hörnlein, International Series of Numerical Mathematics, Birkhäuser, Germany, 1992.

[22]Parkinson, A., and Wilson, M., "Development of a Hybrid SQP-GRG algorithm for Constrained Nonlinear Programming," Design Engineering Technical Conf., ASME Paper 86-DET-114, Columbus, OH, Oct. 1986.

[23]Vanderplaats, G. N., "CONMIN—A FORTRAN Program for Constrained Function Minimization," NASA TM X-62.282, Aug. 1973.

[24]Cook, R. D., *Concepts and Applications of Finite Element Analysis*, Wiley, New York, 1974.

[25]Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T., *Numerical Recipes*, Cambridge Univ. Press, Cambridge, England, UK, 1989.

[26]Jones, R. M., *Mechanics of Composite Materials*, McGraw-Hill, New York, 1975.

[27]Vinson, J. R., and Sierakowski, R. L., *The Behavior of Structures Composed of Composite Materials*, Martinus Nijhoff Publishers, Dordrecht, The Netherlands, 1987.